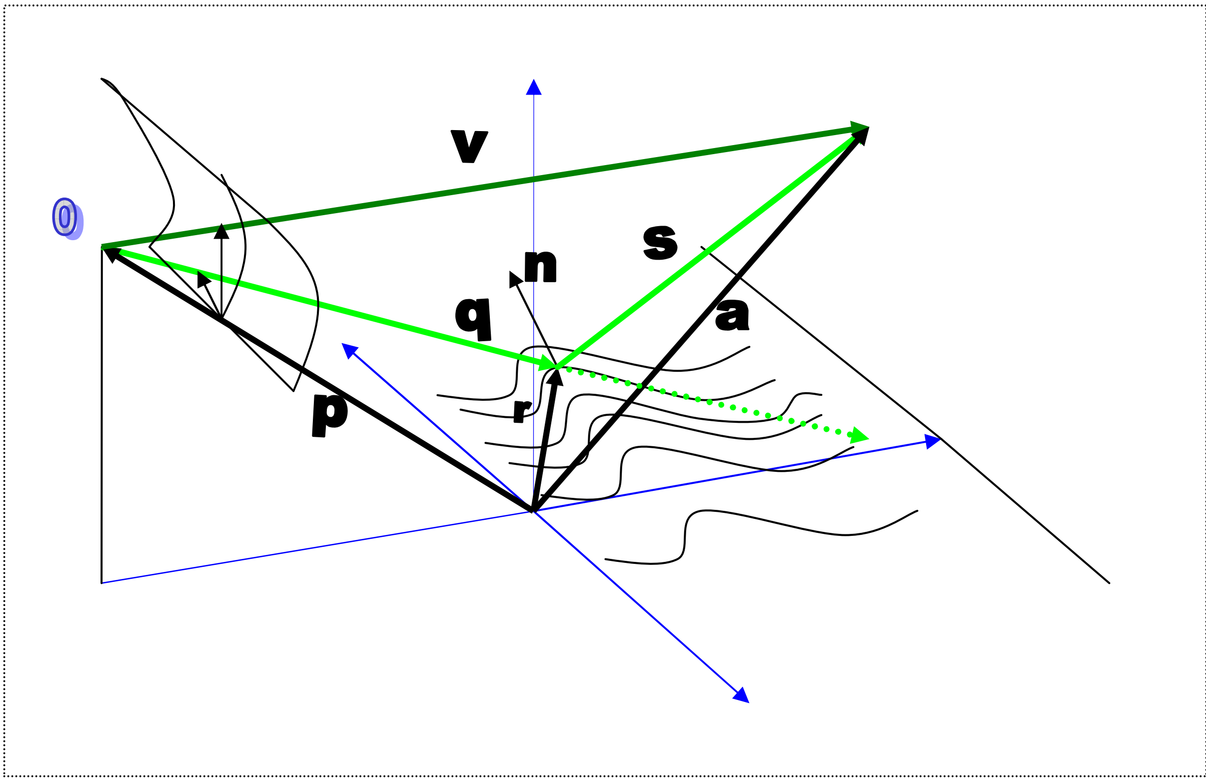
OUTLINE OF REFLECTION THEORY



In the above diagram, the viewer is situated at point **O**, denoted by **p** = (0,-H,V), where H is the HORIZONTAL distance of the viewpoint from the origin, and V is the VERTICAL height. The co-ordinate origin is situated where the blue lines meet. The computer screen is presumed to be the inner surface of a SPHERE, onto which the the lines-of-sight project. Lines-of-sight are coloured in green shades.

ň is the UNIT NORMAL vector to the surface denoted by the wavy lines. The reflection vectors, **q** and **s** lie in the same PLANE as **ň**, and the angle, θ, between the vectors **q** and **ň** is the same as the angle between **s** and **ň** .

The projection onto the computer screen is located by two ANGLES. Firstly, the angle between the vertical plane and the plane containing the vector **q**. Secondly, the angle between the vector **p** and the vector **q**. These angles are then translated into x,y pixel locations on the screen. The .jpg for the sky I use is ALREADY ON THE SCREEN. Therefore, I need to find the screen-colour of the point where the DARK green vector meets the sphere, and use this to decide the screen colour for the point where the LIGHT green vector meets the sphere (computer screen).

v = **q** + k·**ŝ** where **ŝ** denotes the UNIT vector in the direction of **s**. Also, **a** = **r** + k·**ŝ**. The sky is PRESUMED to be situated right at the limit of the array of surface points, say at distance Y along the y-axis. Thus k needs to be set so that the y-coordinate of **a** is y=Y.

Now, a REFLECTION of $\underline{\mathbf{q}}$ in the tangent plane to the surface is achieved by NEGATING the component of $\underline{\mathbf{q}}$ in the direction of $\underline{\check{\mathbf{n}}}$. That is, $\underline{\mathbf{s}}$ is given by;

$$\underline{\mathbf{s}} = \underline{\mathbf{q}} - 2\,(\underline{\mathbf{q}}\cdot\underline{\check{\mathbf{n}}})\underline{\check{\mathbf{n}}}$$

To find k, require that $Y = y + k\{\ (y+H) - 2(\underline{\mathbf{q}}.\underline{\check{\mathbf{n}}})n_y\ \}/\ |\underline{\mathbf{s}}|$,

where n_y is the y-component of $\underline{\check{\mathbf{n}}}$.

Thus $k = (Y-y)\ |\underline{\mathbf{s}}|\ /\ (y + H - 2(\underline{\mathbf{q}}.\underline{\check{\mathbf{n}}})n_y)$, which is needed to get the colour of the sky.

Next, we need to find the two angles determining the point where the LIGHT green line meets the sphere. He first angle, between the vertical plane and the plane containing all the vectors $\underline{\mathbf{p}}$,$\underline{\mathbf{a}}$,$\underline{\mathbf{q}}$,$\underline{\mathbf{r}}$ and $\underline{\mathbf{s}}$, is given by;

$\cos\varphi = (-1,0,0)\cdot(\underline{\mathbf{p}} \wedge \underline{\mathbf{a}})\ /\ |\ \underline{\mathbf{p}} \wedge \underline{\mathbf{a}}\ |$, where (-1,0,0) is the normal to the vertical plane.

and the angle between $\underline{\mathbf{p}}$ and $\underline{\mathbf{q}}$ is given by ;

$\cos\theta = (-\underline{\mathbf{p}})\cdot\underline{\mathbf{q}}\ /\ |\underline{\mathbf{p}}||\underline{\mathbf{q}}|$

That’s it! Admittedly, the algebra is a little more involved when you come to do the actual calculations, but the theory is as above.

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