The Quintessence of Fractals
(avoiding the mathematical intricacies)

The standard way to produce a Fractal on a PC is to use an IFS or Iterated Function System. This is just a jargonised way of saying, in essence, that you take a basic shape, such as a square, and you do something to it, such as rotate it and shrink it – and then display it probably on top of the original. Then you do the same thing again but this time to the rotated, shrunken square (and it'll obviously become more rotated and more shrunken) – and again display it on top of what's already on the screen. Then you do the same again, and again, and again as many times as you decide you want.

The two important points here are that whatever actions you decide to inflict upon the original shape, the result ought to remain within a defined area (the screen would make a sensible choice!) and most importantly it MUST shrink the shape.

Unfortunately, to fully understand the underlying mathematics, you need to think in rather abstract terms. You need to think of each of the shrunken, rotated squares mentioned above as having a label – like a flag stuck in the middle with its name on it; think of them therefore as POINTS. You are then thinking in terms of the SET OF SUBSETS of the governing space – essentially the screen area – and each of the aforementioned flags identifies a member of this set of subsets.

The Contraction Mapping Theorem

The importance of the *shrinking* action mentioned above lies in the Contraction Mapping Theorem, which assures you that so long as the shrinking occurs, then the sequence of "flags" ultimately stop moving around and settle in one spot. That is, the sequence of altered shapes converges to a final altered shape (it doesn't have to be a *point*, but can be a finite sized *shape*, which is all the same a point within the Set of Subsets).

Fractal Dimension: Why Fractals are called Fractals

The term FRACTAL was coined by the mathematician Benoit Mandelbrot in 1975. Many people know that a Fractal is typically a shape or curve which in some sense "looks the same" at different degrees of magnification. This is a typical characteristic of a Fractal, but the term Fractal is meant to indicate that the *Dimension* of the curve is not a whole number. That is, for a Fractal curve drawn on a piece of paper or a screen, it's not of dimension 1, like a straight line, nor of dimension 2, like a rectangle, but in some sense "somewhere in between".

You can estimate the Fractal Dimension of a curve as follows. (I'm deliberately missing out any mathematical justification for doing this; it would need too much background theory.)

- Consider a square 256 x 256 pixels as representing a square 1cm x 1cm, and imagine this square split down firstly into little squares $(1/2^7)$cm or (1/128)th of a cm along the sides. There will be 16384 of these to cover a 1cm x 1cm square. They will also be 2 pixels x 2 pixels in size.
- Additionally, consider the same 1cm x 1cm square covered by slightly larger little squares $(1/2^5)$cm or (1/32)nd of a cm along the sides. These will be 8 x 8 pixels in size.
- Cover the area of the Fractal picture firstly with however many of these larger little squares it takes to just cover the Fractal, say N1.
- Now cover the Fractal picture with however many of the smaller little squares it takes to just cover the Fractal, say N2.
- A calculated estimate for the Fractal Dimension is then given by the formula;

$$\text{Fractal Dimension} = \frac{\ln(N2)-\ln(N1)}{\ln(2^7)-\ln(2^5)} \quad \text{or} \quad \frac{\ln(N2/N1)}{2\ln 2}$$

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